

Consider the following two cases.

**Test A.** We are trying to estimate the mean  $\mu$  of a population's height. We know the standard deviation is  $\sigma = 10$ . We take a sample of size  $N=100$ . We believe the sampling distribution of the mean is normal i.e.  $\bar{X}_A \sim N(\mu, \frac{10}{\sqrt{100}})$  so  $\bar{X}_A \sim N(\mu, 1)$ .

We observe  $\bar{x}_A = 152$ .

We want to calculate the severity with which the claim C:  $\mu > 151$  passes this test with the data we have observed. So we need to calculate  $\Pr(\bar{X}_A \leq 152; \mu \leq 151)$ . We only need to evaluate severity at the point  $\mu = 151$  (because this probability is greater for all values of  $\mu$  less than 151).  $\Pr(\bar{X}_A \leq 152; \mu = 151) = 0.84$  so  $\text{SEV}(T_A, \bar{x}_A=152, C: \mu > 151) = 0.84$ .

**Test B.** We are trying to estimate the mean  $\mu$  of the same population. This time we take a sample of size  $N=10000$ . We believe the sampling distribution of the mean is normal i.e.  $\bar{X}_B \sim N(\mu, \frac{10}{\sqrt{10000}})$  so  $\bar{X}_B \sim N(\mu, \frac{1}{10})$ .

We observe  $\bar{x}_B = 151.1$ .

We want to calculate the severity with which the claim C:  $\mu > 151$  passes this test with the data we have observed.  $\Pr(\bar{X}_B \leq 151.1; \mu = 151) = 0.84$  so  $\text{SEV}(T_B, \bar{x}_B=151.1, C: \mu > 151)=0.84$ .

What I am a little troubled by (just a little!) is the following:

The severity with which claim C passes test A with data  $\bar{x}_A$  is the same as the severity with which claim C passes test B with data  $\bar{x}_B$   
i.e.  $\text{SEV}(T_A, \bar{x}_A=152, C: \mu > 151) = \text{SEV}(T_B, \bar{x}_B=151.1, C: \mu > 151)$ .

However, (I think) the following is true:

$$\Pr(\bar{X}_B \leq 151.1; \mu = a) > \Pr(\bar{X}_A \leq 152; \mu = a)$$

for all  $a < 151$ .

So intuitively because of this fact (if it's true) it seems that the severity with which claim C passes Test B with data  $\bar{x}_B$  is higher than the severity with which claim C passes Test A with data  $\bar{x}_A$ . But perhaps my intuitions are wrong and even if they are right I don't think there would be a way to cash out this intuition in a satisfactory way. But I was just wondering what you thought about this! Thank you again!